Harvard Undergraduate Science Olympiad India 2024
Open Round
Mathematics Syllabus: 9th-10th Grade

Potential Topics Covered on the Exam:
This syllabus contains topics which will be covered both in the open and the in person round. Please note that not necessarily every topic on this list will be on the exam, don't get overwhelmed! The syllabus is meant to be exhaustive of all *potential* topics that could be on the exam. A great place to start is with making sure you're comfortable with the ICSE curriculum for 9th-10th grade. It will be a difficult exam, but remember you don't need, nor do we expect you, to get a 100%! Just do your best and show us all that you've learned!

The topics listed below build off of the syllabus for the open exam, however topics that were formerly listed in red as advanced topics that would not be emphasized on the exam will appear more on the final exam.

Algebra:
- Polynomials:
  - Divisibility and Factorization of Polynomials;
  - Vieta's Theorem;
  - Bezout's Theorem;
  - Binomial Theorem;
  - Rational Root Theorem;
- Sequences and Series:
  - Arithmetic, Geometric, and Other Telescoping Series;
  - Linear Recurrence Relations and their Characteristic Polynomials;
- Inequalities:
  - QM-AM-GM-HM Inequalities;
  - Cauchy-Schwarz Inequality;

- - Triangle Inequality;
  - Trigonometric inequalities;
  - Jensen's Inequality;
- Complex Numbers:
  - Unit Circle;
  - DeMoivre's Theorem;
  - Roots of Unity;
  - Euler's Identity;
- Functions:
  - Injectivity and Surjectivity;
  - Functional Equations;
  - Odd and Even Functions;
  - Concave and Convex Functions:
  - Floor and Ceiling Functions with their properties;
  - Logarithmic and Trigonometric Functions with their properties;

Number Theory:
- Modular Arithmetic - Introduction:
  - Congruences;
  - Prime factorization;
  - Fermat's Little Theorem;
  - Euler's Totient Theorem;
  - Wilson's Theorem;
  - Lagrange's Theorem;
  - Chinese Remainder Theorem;
- Modular Arithmetic - Advanced:
  - Primitive Roots;
  - Quadratic Residues;
  - P-adic valuation and Lifting the Exponent Lemma;
- Miscellaneous:
  - Pythagorean Triples;
  - Diophantine Equations;
  - Number Bases;

Geometry:
- Triangles:
  - Law of Sines;
  - Law of Cosines;
  - Centers of Triangles:
    - Centroid (medians);
    - Circumcenter (perpendicular bisectors);
    - Orthocenter (altitudes);

- - - Incenter (angle bisectors);
    - Excenter (exterior angle bisectors);
  - Area Formulas:
    - Heron's formula;
    - Inradius formula;
    - Circumradius formula;
    - Sine formula;
  - Ratio of Side Lengths:
    - Similar Triangles;
    - Congruent Triangles;
    - Angle Bisector Theorem;
    - Ceva's Theorem;
    - Menelaus' Theorem;
    - Stewart's Theorem;
  - Special Triangles:
    - 3-4-5 (side lengths);
    - 13-14-15 (side lengths);
    - 30-60-90 (angles);
    - 45-45-90 (angles);
- Circles:
  - Tangent Circles;
  - Power of a Point;
  - Cyclic Quadrilaterals;
  - Ptolemy's Theorem;
  - Inscribed Angles;
- Trigonometry:
  - Sum-to-product;
  - Product-to-sum
  - Double-angle;
  - Half-angle;
- Miscellaneous:
  - Coordinate Geometry;
  - 3D Geometry:
    - Volumes of prisms, pyramids, cones, cylinders, cones, and spheres;
    - Surface Areas of prisms, pyramids, cones, cylinders, cones, and spheres;

Combinatorics:
- Counting Principles:
  - Pigeonhole Principle;
  - Invariance Principle;
  - Extremal Principle;

- ○ Coloring Principle;
- ○ Principle of Inclusion-Exclusion;
- ● Enumerative Combinatorics:
  - ○ Permutations, Combinations, and Partitions;
  - ○ Pascal's Triangle;
  - ○ Stars and Bars;
  - ○ Generating Functions;
- ● Probability:
  - ○ Unconditional and Conditional Probability;
  - ○ Distributions;
  - ○ Expectation and Variance;
  - ○ Geometric Probability;
- ● Strategies:
  - ○ Casework;
  - ○ Constructive Counting;
  - ○ Complementary Counting;
  - ○ Finding Bijections;
  - ○ Symmetry;

Preparation for Exam: The AOPS community website provides examples of textbooks as well as problems with their solutions which can be used to study for mathematics olympiads. Also, the following books which can be found online cover good fundamentals.
- Problem-Solving Strategies by Arthur Engel;
- The Arts of Crafts of Problem Solving by Paul Zeitz.

Note that these books and websites are not required to prepare for the exam, nor are they the only way possible to prepare. We expect the difficulty and style of the questions in this exam to resemble those of the questions in competitions like AMC10, AMC12, AIME, HMMT, etc. Therefore, check out the questions and their solutions from these exams on the AOPS community website: https://artofproblemsolving.com/community.